# Product Market Competition and Corporate Financial Policy 

Sudipto Dasgupta<br>Chinese University of Hong Kong and CEPR

SAIF Workshop, July 14, 2019

## Competition and Leverage

- Large literature on how capital structure choices and product market choices (e.g. prices, quantities, or price-cost margins) of competing firms are jointly determined in equilibrium.
- Brander-Lewis (1986), Maksimovic (1998), Showalter (1995), Chevalier and Scharfstein (1995), Dasgupta and Titman (1998)
- We will first review some of these models.


## Quantity Competition - Simple Duopoly with

## Differentiated Products

Start with the system of linear demand functions

$$
\left\{\begin{array}{l}
p_{1}=a-q_{1}-b q_{2} \\
p_{2}=a-q_{2}-b q_{1}
\end{array}\right.
$$

where $b<1$. Assume for simplicity that marginal cost is constant and $c$ for both firms. Cost function of firm $i$ is:

$$
C\left(q_{i}\right)=c q_{i}+k
$$

where $k$ represents fixed costs. Then firm i's profit is given by:

$$
\pi^{i}=\left(p_{i}-c\right) q_{i}-k=\left(a-q_{i}-b q_{j}-c\right) q_{i}-k=R^{i}\left(q_{i}, q_{j}\right)
$$

When profit functions are written in this form, the underlying assumption is that firms are choosing quantities to maximize profits.

## Quantity Competition

Maximizing firm i's profit

$$
R^{i}\left(q_{i}, q_{j}\right)=\left(p_{i}-c\right) q_{i}-k=\left(a-q_{i}-b q_{j}-c\right) q_{i}-k
$$

w.r.t $q_{i}$ gives the first-order condition:

$$
a-2 q_{i}-b q_{j}-c=0
$$

or

$$
q_{i}=\frac{a-c}{2}-(b / 2) q_{j}
$$

which gives firm $i$ 's reaction function. Similarly, firm $j$ 's reaction function is

$$
q_{j}=\frac{a-c}{2}-(b / 2) q_{i}
$$

Note that the reaction functions are negatively sloped: quantities are strategic substitutes.


## Firm-Specific Uncertainty

We will introduce uncertainty and work with profit functions of the form $R^{i}=R^{i}\left(q_{i}, q_{j}, z_{i}\right)$ where $z_{i}$ and $z_{j}$ are i.i.d random variables distributed with $\operatorname{cdf} F(z)$.

Here are two (equivalent) ways in which uncertainty can be introduced. (i) $z_{i}$ is a positive shock to the demand intercept. $z_{i}$ is a negative shock to marginal cost.

Under quantity competition, the profit functions in these two cases are:

$$
\begin{aligned}
& R^{i}\left(q_{i}, q_{j}, z_{i}\right)=(\underbrace{p\left(q_{i}, q_{j}\right)+z_{i}}-c_{i}) q_{i} \\
& R^{i}\left(q_{i}, q_{j}, z_{i}\right)=(p\left(q_{i}, q_{j}\right)-\underbrace{\left(c_{i}-z_{i}\right)}) q_{i}
\end{aligned}
$$

## Brander \& Lewis, 1986

Two firms form duopoly in the market decide their debt level at time 0 , decide output level at time 1 and then at time 2, firms pay back debt after randomness is realized.

| $t=0$ | $t=1$ | $t=2$ |
| :--- | :--- | :--- |
| firms issue debt | firms choose output | debt due, <br> period 2 profit realized |

## Brander \& Lewis contd.

Profit function $R^{i}\left(q_{i}, q_{j}, z_{i}\right), z_{i}, z_{j}$ i.i.d random with c.d.f. $F\left(z_{i}\right)$,

$$
\text { (i) } R_{i i}^{i}<0 \text {, (ii) } R_{j}^{i}<0 \text {, (iii) } R_{i j}^{i}<0 \text {, (iv) } R_{z}^{i}>0, \text { (v) } R_{i z}^{i}>0
$$

Note that for our linear example:

$$
\begin{gathered}
R^{i}\left(q_{i}, q_{j}, z_{i}\right)=\left(p\left(q_{i}, q_{j}\right)+z_{i}-c_{i}\right) q_{i}, \text { and } \\
p\left(q_{i}, q_{j}\right)=a-q_{i}-b q_{j} \\
R_{i}^{i}=p\left(q_{i}, q_{j}\right)+z_{i}-c_{i}+p_{1} q_{i}, \text { and } \\
R_{i i}^{i}=2 p_{1}+p_{11} q_{i}=-2<0, R_{j}^{i}=p_{2} q_{i}=-b q_{i}<0, \\
R_{i j}^{i}=p_{2}+p_{12} q_{i}=-b<0, R_{z}^{i}=q_{i}>0 .
\end{gathered}
$$

## Brander \& Lewis, contd.

At time 1, problem for firm i is

$$
\begin{aligned}
& \max _{q_{i}} \int_{\hat{z}}^{\bar{z}}\left[R^{i}\left(q_{i}, q_{j}, z_{i}\right)-D_{i}\right] f\left(z_{i}\right) d z_{i}=V^{i} \\
& R^{i}\left(q_{i}, q_{j}, \hat{z}_{i}\right) \equiv D_{i} \\
& \text { FOC: } V_{i}^{i}=\int_{\hat{z}_{i}}^{\bar{z}} R_{i}^{i}\left(q_{i}, q_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}=0 \\
& \text { SOC: } V_{i i}^{i}<0 \\
& V_{i}^{i}\left(q_{i}, q_{j}, D_{i}, D_{j}\right)=0, V_{i i}^{i} d q_{i}+V_{i D} d D_{i}=0 \\
& \frac{d q_{i}}{d D_{i}}=\frac{V_{i D}^{i}}{-V_{i i}^{i}} \\
& V_{i D}^{i}=V_{i \hat{z}_{i}} \frac{d \hat{z}_{i}}{d D_{i}}=-R_{i}^{i}\left(q_{i}, q_{j}, \hat{z}_{i}\right) f\left(\hat{z}_{i}\right) \frac{1}{R_{z}^{i}\left(\hat{z}_{i}\right)}
\end{aligned}
$$

## Brander \& Lewis, contd.

- From FOC and $R_{i z}^{i}>0$, we know $R_{i}^{i}\left(q_{i}, q_{j}, \hat{z}_{i}\right)<0$.
- So it follows that $\frac{d q_{i}}{d D_{i}}>0$.
- Firms i's reaction function will move rightward when it increases debt level.
- As a result, firm i's output will increase while firm j's output will decrease.
- This will allow firm i to move to its optimal point on firm j's reaction function.
- Optimality requires that i's iso-profit line will be tangent to j's reaction function.
- In equilibrium, both firms will choose debt and produce more than if neither firm had any debt.



## $t=0$ Problem

At time $t=0$, firms choose debt to maximize firm value, which is the sum of debt value and equity value.

Equity value is the (present value of) cash flows to equity holders after debt is chosen, given by $V$ above.
Recall that $V^{i}=\int_{\hat{z}_{i}}^{\bar{z}}\left[R^{i}\left(q_{i}, q_{j}, z_{i}\right)-D_{i}\right] f\left(z_{i}\right) d z_{i}$.
Debt value, also called the market value of debt, is the amount the firm can raise in the market after pledging $D$.

If the firm is solvent, i.e., $z \geq \hat{z}$, the debt is paid in full (recall:
$R^{i}\left(q_{i}, q_{j}, \hat{z}_{i}\right)=D_{i}$ and $R_{z}^{i}>0$.
If the firm is insolvent, $z<\hat{z}_{i}$, debtholders only get
$\left(R^{i}\left(q_{i}, q_{j}, z_{i}\right)<D_{i}\right.$.
Therefore, Debt value is

$$
D_{0}^{i}=\int_{\underline{z}}^{\hat{z}_{i}} R^{i}\left(q_{i}, q_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}+\left(1-F\left(\hat{z}_{i}\right)\right) D_{i} .
$$

## $t=0$ Problem

$$
\begin{aligned}
\operatorname{Max}_{D_{i}} & Y^{i}\left(D_{i}, D_{j}\right)=V_{i}\left(D_{i}, D_{j}\right)+D_{0}^{i}\left(D_{i}, D_{j}\right) \\
= & \int_{\hat{z}_{i}}^{\bar{z}}\left[R^{i}\left(q_{i}, q_{j}, z_{i}\right)-D_{i}\right] f\left(z_{i}\right) d z_{i} \\
+ & \int_{\underline{z}}^{\hat{z}_{i}} R^{i}\left(q_{i}, q_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}+\left(1-F\left(\hat{z}_{i}\right)\right) D_{i} \\
= & \int_{\underline{z}}^{\bar{z}} R^{i}\left(q_{i}, q_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}
\end{aligned}
$$

Note that since firms chose $q_{i}$ and $q_{j}$ after $D_{i}$ and $D_{j}$ are chosen, $q_{i}=q_{i}\left(D_{i}, D_{j}\right)$ and $q_{j}=q_{j}\left(D_{j}, D_{i}\right)$.

## $t=0$ Problem

$$
\begin{aligned}
& \frac{\partial Y^{i}\left(D_{i}, D_{j}\right)}{\partial D_{i}}=\frac{\partial \int_{\underline{z}}^{\bar{z}} R^{i}\left(q_{i}, q_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}}{\partial D_{i}} \\
& \left.\quad=\left[\int_{\hat{z}_{i}}^{\bar{z}} R_{i}^{i}\left(q_{i}, q_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}\right] \frac{\partial q_{i}}{\partial D_{i}}\right] \\
& \left.\quad+\left[\int_{\underline{z}}^{\hat{z}} R_{i}^{i}\left(q_{i}, q_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}\right] \frac{\partial q_{i}}{\partial D_{i}}\right] \\
& \left.\quad+\left[\int_{\underline{z}}^{\bar{z}} R_{j}^{i}\left(q_{i}, q_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}\right] \frac{\partial q_{j}}{\partial D_{i}}\right]
\end{aligned}
$$

The first bracketed term is zero, the second is negative and the third is positive. However, for $D_{i} \rightarrow 0, \hat{z}_{i} \rightarrow \underline{z}$, so the second term $\rightarrow 0$. Hence

$$
\left.\frac{\partial Y^{i}\left(D_{i}, D_{j}\right)}{\partial D_{i}}\right|_{D_{i}=0}>0
$$

## Price Competition

Refer back to the linear demand functions we started with. Now, rewrite the demand functions as follows:

$$
\left.\begin{array}{l}
\left\{\begin{array}{l}
q_{1}=\alpha-\beta p_{1}+r p_{2} \\
q_{2}=\alpha-\beta p_{2}+r p_{1}
\end{array}\right. \\
\alpha=\frac{a}{1+b}, \beta=\frac{1}{1-b^{2}}, r=\frac{b}{1-b^{2}}
\end{array}\right\} \begin{aligned}
& \pi_{1}=\left(p_{1}-c\right)\left(\alpha-\beta p_{1}+r p_{2}\right)-k=R\left(p_{1}, p_{2}\right) \\
& \max _{p_{1}} R\left(p_{1}, p_{2}\right) \\
& \text { FOC: } \alpha-\beta p_{1}+r p_{2}-\beta\left(p_{1}-c\right)=0
\end{aligned}
$$

1 's reaction function $p_{1}=\frac{\alpha}{2 \beta}+\frac{r}{2 \beta} p_{2}+\frac{c_{1}}{2}$
noting that the reaction function has positive slope, we conclude that prices are strategic complements.


## Price Competition contd.

Redo Brander\& Lewis: Assume

$$
\begin{aligned}
& R^{1}\left(p_{1}, p_{2}, z_{1}\right), R_{i i}^{i}<0, R_{j}^{i}>0, R_{i j}^{i}>0, R_{z}^{i}>0, R_{i z}^{i}>0(?) \\
& \max _{p_{i}} \int_{\hat{z}_{i}}^{\bar{z}}\left[R^{i}\left(p_{i}, p_{j}, z_{i}\right)-D_{i}\right] f\left(z_{i}\right) d z_{i}=V^{i} \\
& \text { with } R^{i}\left(p_{i}, p_{j}, \hat{z}_{i}\right) \equiv D_{i}
\end{aligned}
$$

$$
\text { FOC: } V_{i}^{i}=\int_{\hat{z}_{i}}^{\bar{z}} R_{i}^{i}\left(p_{i}, p_{j}, z_{i}\right) f\left(z_{i}\right) d z_{i}=0
$$

$$
V_{i i} d p_{i}+V_{i D} d D_{i}=0, \frac{d p_{i}}{d D_{i}}=\frac{V_{i D}}{-V_{i i}}
$$

$$
V_{i D}=V_{i \hat{z}_{i}} \frac{d \hat{z}_{i}}{d D_{i}}=-R_{i}\left(p_{i}, p_{j}, \hat{z}_{i}\right) f\left(\hat{z}_{i}\right) \frac{1}{R_{z}\left(\hat{z}_{i}\right)}
$$

If $R_{i z}>0$, by the same argument as before, $\frac{d p_{i}}{d D_{i}}>0$.


## Price Competition

- Hence, firm i's reaction function shifts out.
- Since reaction functions are positively sloped, the new equilibrium price is higher for both firms.
- This seems consistent with Chevalier's (1995a, 1995b) empirical results.
- How robust is this result?


## Robustness of B \& L Contd.

Consider two types of uncertainty introduced earlier.
Note that under both situations, profits will increase in $z_{i}$, i.e. $R_{z_{i}}^{i}>0$.
Under quantity competition, the profit functions in these two cases are:

$$
\begin{aligned}
& R\left(q_{i}, q_{j}, z_{i}\right)=\left(p\left(q_{i}, q_{j}\right)+z_{i}-c_{i}\right) q_{i}, \quad R_{z}=q_{i}, \quad R_{z i}=1 \\
& R\left(q_{i}, q_{j}, z_{i}\right)=\left(p\left(q_{i}, q_{j}\right)-\left(c_{i}-z_{i}\right)\right) q_{i}, R_{z}=q_{i}, R_{z i}=1
\end{aligned}
$$

So the assumption of $R_{i z}>0$ holds.

## Robustness contd.

Under price competition, we have

$$
\begin{aligned}
& R\left(p_{i}, p_{j}, z_{i}\right)=\left(p_{i}-c_{i}\right)\left(q\left(p_{i}, p_{j}\right)+z_{i}\right), R_{z}=p_{i}-c_{i}, R_{z i}=1 \\
& R\left(p_{i}, p_{j}, z_{i}\right)=\left(p_{i}-\left(c_{i}-z_{i}\right)\right) q\left(p_{i}, p_{j}\right), \\
& R_{z}=q\left(p_{i}, p_{j}\right), R_{z i}=\frac{\partial q}{\partial p_{i}}<0 .
\end{aligned}
$$

Thus, when the uncertainty is from the cost side, $R_{i z}<0$, and debt will cause the reaction functions to shift in, and lead to lower equilibrium prices.

## Takeaways from B \& L

- With quantity competition, firm becomes more aggressive with debt when the randomness is from either demand or cost.
- With price competition, firm becomes less aggressive with debt when the randomness is from demand, but more aggressive with debt when the uncertainty is from cost.
- Unless the nature of competition or source of uncertainty can be determined, difficult to test these models.


## Multi-period Models: Glazer, 1994

| time 0 | time 1 | time2 |
| :--- | :--- | :--- |
| debt issued, <br> period 1 output chosen | period 2 <br> output chosen | debt due, <br> period 2 profit realized |

Net debt for firm $i$ at $t=1$ (beginning of second period, prior to choosing second period output) is $D_{i}-\pi_{i}$. Firm $j$ will lower its period 1 output (less aggressive) to increase $\pi_{i}$ so that firm $i$ is less aggressive in period 2.

Debt issuance leads to lower output and higher period 1 price even under quantity competition.

## Repeated Oligopoly; Maksimovic, 1988

Consider an n-firm symmetric Oligopoly.
Under collusion, each firm produces $1 / n$ of the monopoly output. Let $q^{c}$ denote the output of each firm, and $\pi^{c}$ the corresponding collusive profit.

Assume that if any firm deviates, for all subsequent periods, the industry equilibrium is characterized by the "trigger strategy"
outcome of each firm producing the Cournot-Nash equilibrium output $q^{n c}$, with the corresponding profit being $\pi^{n c}$.

Let $r$ be the discount rate, and let $\pi^{d}>\pi^{c}$ denote the deviation profit, i.e., the profit to the deviating firm in the deviation period.

## Maksimovic, contd.

Collusion is sustainable if: $\pi^{d}+\frac{\pi^{n c}}{r}<\pi^{c}+\frac{\pi^{c}}{r} \quad(*)$.
Assume the above condition holds and let a firm choose debt that pays interest $b$ in perpetuity. Suppose $b>\pi^{n c}$.

If the firm deviates, assume it responds to other firms' $q^{n c}$ by playing $q^{n c}$ as well. It will immediately default since $b>\pi^{n c}$. After the first period after defection, the firm is with the new owners - creditors - and they will also play $q^{n c}$.

Defection will occur if $\pi^{d}-b>\pi^{c}-b+\frac{\pi^{c}-b}{r}$.
Rewriting, we get: $\pi^{d}+\frac{b}{r}>\pi^{c}+\frac{\pi^{c}}{r} \quad(* *)$.
For $b$ sufficiently high, both conditions (*) and (**) can hold, and debt can trigger a breakdown of collusion.

## Switching Cost Models

Two firms ( $A$ and $B$ ) and 2 periods ( $i=1,2$ ).
Let $x_{i}$ denote profit in period $i, i=1,2$.
$\sigma_{A}$ and $\sigma_{B}$ denote period 1 market shares of the firms, respectively:
$\sigma_{A}+\sigma_{B}=1$.
Period 1 prices determine period 1 profits and period 1 market shares for firm $A$ (and similarly for firm $B$ ):

$$
x_{1}^{A}=x_{1}\left(P_{1}^{A}, p_{1}^{B}\right), \sigma_{A}=\sigma_{A}\left(P_{1}^{A}, p_{1}^{B}\right) .
$$

Period 1 market shares affect period 2 profits:

$$
x_{2}^{A}=x_{2}\left(\sigma_{A}, \sigma_{B}, P_{2}^{A}, P_{2}^{B}\right) .
$$

In period two, firms $A$ and $B$ choose $P_{2}^{A}$ and $P_{2}^{B}$, respectively, to maximize $x_{2}^{A}\left(\sigma_{A}, \sigma_{B}, P_{2}^{A}, P_{2}^{B}\right)$ and $x_{2}^{B}\left(\sigma_{A}, \sigma_{B}, P_{2}^{A}, P_{2}^{B}\right)$, so
$P_{2}^{A}=P_{2}^{A}\left(\sigma_{A}, \sigma_{B}\right)$ and $P_{2}^{B}=P_{2}^{B}\left(\sigma_{A}, \sigma_{B}\right)$

## Switching Cost Models contd.

The period 1 problem for firm $A$, for example, thus becomes

$$
\operatorname{Max}_{P_{1}^{A}} \quad x_{1}^{A}\left(P_{1}^{A}, P_{1}^{B}\right)+x_{2}^{A}\left(\sigma_{A}, \sigma_{B}, P_{2}^{A}\left(\sigma_{A}, \sigma_{B}\right), P_{2}^{B}\left(\sigma_{A}, \sigma_{B}\right)\right)
$$

Since $\sigma_{A}$ and $\sigma_{B}$ are functions of $P_{1}^{A}$ and $P_{1}^{B}$ and $\sigma_{A}+\sigma_{B}=1$, we can write the problem as

$$
\operatorname{Max}_{P_{1}^{A}} \quad x_{1}^{A}\left(P_{1}^{A}, P_{1}^{B}\right)+x_{2}^{A}\left(\sigma_{A}\left(P_{1}^{A}, P_{1}^{B}\right)\right)
$$

Assume $\frac{\partial x_{2}^{A}}{\partial \sigma_{A}}>0, \frac{\partial \sigma_{A}}{\partial P_{1}^{A}}<0$ and $\frac{\partial \sigma_{A}}{\partial p_{1}^{B}}>0$.

$$
\text { FOC: } \frac{\partial x_{1}^{A}}{\partial P_{1}^{A}}+\frac{\partial x_{2}^{A}}{\partial \sigma_{A}} \frac{\partial \sigma_{A}}{\partial P_{1}^{A}}=0
$$

so $\frac{\partial x_{1}^{A}}{\partial P_{1}^{A}}>0$, the slope is positive on the $\left(x_{1}, P_{1}\right)$ diagram, suggesting investment in market share.

## Dasgupta and Titman, 1998

| $t=0$ | $t=1$ | $t=2$ |
| :--- | :--- | :--- |
| debt issued; | $x_{1}$ realized; | debt due, $x_{2}$ realized; |
| period 1 | $I-x_{1}$ new junior debt issued; | liquidation value $\tilde{L}$ |
| price chosen | period 2 <br> price chosen | realized; <br> debt repayment |

$I$ is a fixed investment need at $t=1$. It could be either regarded as replacement investment with no additional cash flow implications, or one needs to assume that the cash flow from this investment cannot be pledged to debt holders.
$I-x_{1}$ has to be financed by issuing new junior debt due at date 2 .
$\tilde{L} \sim F()$.
Face value of junior debt is $y$ and that of senior debt is $d$.

Start with period $2(t=1)$. The equity value is

$$
\begin{aligned}
& E_{1}=\int_{d+y-x_{2}}^{\bar{L}}\left(x_{2}+\tilde{L}-d-y\right) d F \\
& \text { FOC: } \frac{\partial}{\partial P_{2}^{A}} \int_{d+y-x_{2}}^{\bar{L}}\left(x_{2}+\tilde{L}-d-y\right) d F=0 . \\
& \Rightarrow \int_{d+y-x_{2}}^{\bar{L}} \frac{\partial x_{2}^{A}}{\partial P_{2}^{A}} d F=0 \\
& \Rightarrow \frac{\partial x_{2}^{A}}{\partial P_{2}^{A}}=0 .
\end{aligned}
$$

Hence the functions $P_{2}^{A}=P_{2}^{A}\left(\sigma_{A}, \sigma_{B}\right)$ and $x_{2}^{A}=x_{2}^{A}\left(\sigma_{A}\left(P_{1}^{A}, P_{1}^{B}\right)\right)$
are the same as in the all-equity case.

Now consider the time $t=0$ problem. The equity value after $d$ is chosen but before $I-x_{1}$ is raised is

$$
\begin{aligned}
E_{0} & =\int_{d+y-x_{2}}^{\bar{L}}\left(x_{2}+\tilde{L}-d-y\right) d F \\
& =\int_{d+y-x_{2}}^{\bar{L}}\left(x_{2}+\tilde{L}-d\right) d F-y\left(1-F\left(d+y-x_{2}\right)\right)
\end{aligned}
$$

Now the face value of the junior debt $y$ is given by

$$
I-x_{1}=y\left[1-F\left(d+y-x_{2}\right)\right]+\int_{d-x_{2}}^{d+y-x_{2}}\left(\tilde{L}+x_{2}-d\right) d F
$$

Substituting, we get

$$
E_{0}=\int_{d-x_{2}}^{\bar{L}}\left(x_{2}+\tilde{L}-d\right) d F+x_{1}-\jmath
$$

Maximizing $E_{0}$ with respect to $P_{1}^{A}$ gives the time 0 FOC:

$$
\frac{\partial x_{2}^{A}}{\partial P_{1}^{A}}\left(1-F\left(d-x_{2}\right)\right)+\frac{\partial x_{1}^{A}}{\partial P_{1}^{A}}=0
$$

## Comparison with All-Equity Case

All Equity FOC:

$$
\frac{\partial x_{2}^{A}}{\partial P_{1}^{A}}+\frac{\partial x_{1}^{A}}{\partial P_{1}^{A}}=0 .
$$

With risky debt:

$$
\frac{\partial x_{2}^{A}}{\partial P_{1}^{A}}\left(1-F\left(d-x_{2}\right)\right)+\frac{\partial x_{1}^{A}}{\partial P_{1}^{A}}=0 .
$$

For given $P_{1}^{B}$, debt leads to higher $P_{1}^{A}$ : firm $A$ 's reaction function shifts out.

Since prices are strategic complements, this causes both $P_{1}^{A}$ and $P_{1}^{B}$ to increase.

Evidence very consistent with Chevalier (1995).

## Optimality of Debt

$$
\begin{aligned}
& d_{0}=d\left(1-F\left(d-x_{2}\right)\right)+\int_{0}^{d-x_{2}}\left(x_{2}+\tilde{L}\right) d F \quad \text { market value of debt } \\
& E_{0}=\int_{d-x_{2}}^{\bar{L}}\left(x_{2}+\tilde{L}-d\right) d F+x_{1}-I \\
& V=d_{0}+E_{0}=x_{2}+x_{1}+\int_{0}^{\bar{L}}(\tilde{L}) d F-I \\
& \frac{d V}{d(d)}=\left[\frac{\partial x_{1}^{A}}{\partial P_{1}^{A}}+\frac{\partial x_{2}^{A}}{\partial \sigma_{A}} \frac{\partial \sigma_{A}}{\partial P_{1}^{A}}\right] \frac{d P_{1}^{A}}{d(d)}+\left[\frac{\partial x_{1}^{A}}{\partial P_{1}^{B}}+\frac{\partial x_{2}^{A}}{\partial \sigma_{A}} \frac{\partial \sigma_{A}}{\partial P_{1}^{B}}\right] \frac{d P_{1}^{B}}{d(d)}
\end{aligned}
$$

If $d=0$, the first part is zero $\left(\frac{\partial x_{1}^{A}}{\partial P_{1}^{A}}+\frac{\partial x_{2}^{A}}{\partial \sigma_{A}} \frac{\partial \sigma_{A}}{\partial P_{1}^{A}}=0\right)$.
The second part is positive, so $d_{0}=0$ is not optimal.
Hence, some debt will be chosen in equilibrium.

